Week 1 Problems

MathMagic Mentors

June 2024

1 Prealgebra

1.1 Arithmetic

Try these without a calculator!

1. What is $1 - 2 + 0 \div 3 + 3 \times 7$?
2. Calculate $9 + 8 \cdot (7 + 6) + (5 - 4) \cdot 3 - 2 \cdot 1$.
3. Find $-(11)(7) + (-10)(10)$.
4. Let $x = 2$ and $y = 5$. What is $2x + 5y^2 - 3xy$?
5. Compute $3^3 + 2^0 - 7 + 5$.
6. Find the value of the following expression if $x = 3$:
$$2x^3 - 5x^2 + 7$$
7. What is the value of $24 \cdot 5 \cdot 100 \cdot 2$?
8. Express $\frac{1}{7} + \frac{1}{8}$ as a simplified common fraction.
9. Find $2 \cdot 3 \cdot 25 \cdot 5 \cdot 4 \cdot 10$.
10. Sauron has three nickels, five dimes and five quarters. How much money does he have in cents?
11. Compute
$$117 \cdot 1001 + 9 \cdot 999 \cdot 13.$$
12. Given the expression: $1990 \times 1991 - 1989 \times 1990$. What counting number is equivalent to the expression?
13. Six times the reciprocal of a number is $\frac{9}{4}$. What is the number? Express your answer as a mixed number.

14. Compute the sum

$$51 \cdot 23 + 49 \cdot (-11) + 49 \cdot 11 + 51 \cdot (-23).$$

15. Find $2^2 - 1^2$. Find $3^2 - 2^2$. Find $4^2 - 3^2$. Can you find a pattern and generalize? Why does it work?

2 Algebra I

2.1 Linear Equations

1. Solve for x: $5x - 3 = 97$.
2. Find the value of x such that $2x + 5 = 3x + 7$.
3. What is the value of x that satisfies

$$2(5 - x) + 3 = 5(x - 4)?$$

4. Find the value of $2y + 4$ when $3(2y + 4) + 5 = -(2y + 4)$
5. Solve for c: $\frac{c}{7} + \frac{c}{3} = 1 - c$.
6. Find the value of $2a + 6$ when $3(2a + 7) + 42 = -(2a - 9)$
7. The product of two consecutive even integers is 32 less than the square of the greater integer. Find the smaller integer.
8. Daniel has three more than twice as much money in his bank account than he did in May. The sum of his balance in May and June is \$90. How much money does he currently have?
9. Find the value of x that satisfies the equation

$$25^{-2} = \frac{5^{48/x}}{5^{26/x} \cdot 25^{17/x}}.$$

2.2 Linear Inequalities

1. Solve the inequality $3(5 - x) \leq 24$.
2. Find all x so that **neither** $-2(6 + 2x) \leq -16$ **nor** $-3x \geq 18$.
3. How many integers x satisfy $-7 \leq 3x + 1 \leq 5$?
4. Sid is going camping and he needs to spend his budget wisely! He must spend at least ten dollars. Given that an orange costs 1.5 dollars more than an apple and he buys 2 apples and 4 oranges, what is the cost of three oranges?

5. A tennis player computes her win ratio by dividing the number of matches she has won by the total number of matches she has played. At the start of a weekend, her win ratio is exactly 0.500. During the weekend, she plays four matches, winning three and losing one. At the end of the weekend, her win ratio is greater than 0.503. What's the largest number of matches she could've won before the weekend began? (AIME 1992)

2.3 Systems of Equations

1. The sum of the numerator and the denominator of a fraction is 216. The fraction is equivalent to $\frac{2}{7}$. What is the value of the denominator?

2. Solve for x:

$$3x - 7y = 14$$
$$2x + 7y = 6.$$

3. Find the sum $x + 2y$ if

$$3x - y = 11$$
$$6x + y = 7.$$

4. A football game was played between two teams, the Cougars and the Panthers. The two teams scored a total of 34 points, and the Cougars won by a margin of 14 points. How many points did the Panthers score? (Source: AMC 12)

5. Together, Larry and Lenny have \$35. Larry has two-fifths of Lenny's amount. How many more dollars than Larry does Lenny have?

6. Four distinct integers a, b, c and d have the property that when added in pairs, the sums 10, 18, 19, 20, 21, and 29 are obtained. What are the four integers in increasing order? (place a comma and then a space between each integer)

7. Alex, Bob, Camille, and Danielle's mothers are comparing their children's ages. They observe that the sum of Alex, Bob, and Danielle's ages is fourteen times Camille's age. They also note that the sum of Alex and Bob's ages is six times Camille's age, and Bob's age is two years less than the difference in ages of Danielle and Alex. How old is Camille?

8. Solve for the triple (x, y, z) in the following system:

$$2x + y + z = 7$$
$$x + 2y + z = 4$$
$$x + y + 2z = 9$$

3 Algebra II

4 Functions

1. Let $f(x) = 2x + 3$ and $g(x) = 3x + 4$. Find:
   - $f(x) \cdot g(x)$ (also symbolized as $(f \cdot g)(x)$.
   - $f(x) + g(x)$
   - $2f(x) + 3g(x) - 7x^2 + 3$.
   - $f(g(x))$
2. Find the domain of $f(x) = \sqrt{x-3}$. Find its range.
3. What is the range of $g(x) = -x^2 + 6x + 5$?
4. Find the largest real number c such that 1 is in the range of $f(x) = x^2 - 5x + c$.
5. Let $f(x) = ax^2 + bx + c$ where $a > 0$. What is the domain of f? What is the range? Express your answer in interval notation.
6. Find the inverse function f^{-1} of $f(x) = 6x - 5$.
7. Find the inverse function of $f(x) = \sqrt{2x-5}$. State the domain and range of f^{-1}.
8. Find the number of values of x for which the expression $\frac{x^2-9}{(x^2+2x-3)(x-3)}$ is undefined.
9. Let f be a function such that $f(x) = -f(-x)$. Can you find such a function with this property? Graph it. What properties do you notice?
10. Let f be a function such that $f(x) = f(-x)$. Can you find such a function with this property? Graph it. What properties do you notice?
11. If $f(x) = \frac{ax+b}{cx+d}, abcd \neq 0$ and $f(f(x)) = x$ for all x in the domain of f, what is the value of $a + d$?